Course description

Course title	Course code
Applied Actuarial Mathematics (Taikomoji aktuarinė matematika)	

Lecturers	Department where the course is delivered
Assist. Dr. Rokas Gylys **Assist. Dr. Aldona Skučaitė**	Department of Mathematical Analysis Faculty of Mathematics and Informatics Naugarduko St. 24, LT-03225 Vilnius, Lithuania

Cycle	Type of course
Second	Compulsory

Mode of delivery	Semester or period when the course is delivered	Language of instruction
Face-to-face	2nd semester (Spring)	Lithuanian, English

Prerequisites and corequisites	
Prerequisites: Probability Theory and Mathematical Statistics (First level) Survival Demographics Models (First level) Actuarial Mathematics (First level) Non-life insurance models (First level)	**Corequisites (if any):**

Number of ECTS credits	Student's workload	Contact hours	Individual work hours
10	251	88	163

Course objectives: programme competences to be developed

The aim of this course is to acquaint students with actuarial models used in non-life insurance and pensions. **General** competences developed: a) to be able to work independently and as a team member; b) time management and accurate performance of tasks; c) ability to describe what further steps are needed for deeper analysis of problem. **Professional** competences developed: a) to be able to apply mathematical methods for solution of actuarial problems; b) to be able to present results for specialists and wide audience.

This course is partially prepared together with the Lithuanian Actuarial Society. The course focuses on the key tasks encountered by non-life insurance / pension actuaries, such as pricing, reserving, reinsurance, risk and capital management. The theoretical studies of the mathematical models are combined with the case studies of their practical application, including presentations of the leading actuaries of the insurance companies operating in the Baltics.

Learning objectives. At the end of the course a student should be able to:	Learning methods	Assessment methods
– Apply key concepts of non-life insurance. Be able to analyze different reinsurance arrangements. Understand the principles of assessment of solvency at non-life insurance company.	Lectures Demonstration	Final test
– Analyze non-life insurance reserving methods. Apply deterministic claims reserving methods with real life	Lectures Demonstration	Group assignment Final Test

date. Understand the basics of stochastic claims reserving methods.	Problem solving	
– Analyze statistical models for claim amount. Analyze collective and individual risk models. Apply non-life insurance risk models with real life claims data using statistical analysis software.	Lectures Demonstration Problem solving	Group assignment Final Test
– Analyze classical and Bayesian credibility models in non-life insurance context. Apply credibility theory methods with real life claims data.	Lectures Demonstration Problem solving	Group assignment Final Test
– Analyze generalized linear models in non-life insurance context. Apply generalized linear models with real life claims data. Understand the basics of application of artificial neural networks and regression tree-based models for non-life insurance pricing.	Lectures Demonstration Problem solving	Group assignment Final Test
– Explain system of 3 pension pillars and classify pension systems. Understand the concept of "solidarity between generations" and its importance for every pension system. Critically assess pension systems and analyze its advantages and disadvantages. Explain differences of pension systems (demographic, financial, etc.) for wide audience.	Lectures Reading of articles Oral presentation	Short oral presentation about chosen country's pension system.
– Understand / explain demographic and financial model(s) of pension system and mechanism of funding	Lectures Demonstration Problem solving	Individual or group assignment Final Test
– Explain differences between deterministic and stochastic approach to pension annuities; calculate risk measures (of pension annuities).	Lectures Demonstration Problem solving	Individual or group assignment Final Test
– Understand / explain concept of longevity risk, explain differences between cohort and period mortality tables; – Apply main methods of construction of projected mortality tables and explain advantages and disadvantages of different methods. – Assess impact of longevity risk: calculate risk measures; distinguish between pooling and non-pooling risk, etc.; – Explain advantages and disadvantages of main methods of risk management (hedging, reinsurance etc.)	Lectures Demonstration Problem solving	Individual or group assignment Final Test
– Select and apply the appropriate actuarial model considering the purpose, available data, risk profile, and other factors. – Explain and communicate the application of actuarial models and results derived using these models.	Case study Explanation Demonstration Group learning	Individual or group assignment

Course content: breakdown of the course	Contact hours			Individual work hours and assignments	
	Lectures	Practical training	**Total contact hours**	**Individual work hours**	**Assignments**
1. Key concepts of non-life insurance. Actuarial profession. Types of reinsurance arrangements. Regulatory environment.	**3**	**1**	**4**	**4**	Decide on the groups for the group assignment. Read ISAP1 https://www.actuaries.org/CTTEES_ASC/Documents/ReformattedISAP1FINALOCTOBER_correctedJan2014.pdf
2. Non-life insurance reserving. Types of reserves. Chain-ladder and Bornhuetter-Ferguson models. Introduction to non-life insurance stochastic claims reserving models.	**3**	**3**	**6**	**8**	Solve assigned problems. Study non-life technical provisions methods by reading Ch. 1 of [1] or alternative sources (references will be provided).
3. Non-life insurance loss distributions. Collective risk models. Individual risk model. Model selection and goodness-of-fit tests.	**4**	**4**	**8**	**8**	Solve assigned problems. Study the key statistical distributions used in non-life insurance by reading Ch. 2 and 3 of [1] or alternative sources (references will be provided).
4. Credibility theory in non-life insurance. Classical credibility models. Bayesian credibility models.	**3**	**4**	**7**	**8**	Solve assigned problems. Study Ch. 5 of [1] or alternative sources (references will be provided).
5. Non-life insurance pricing using generalized linear models. Introduction to non-life insurance pricing using artificial neural networks and regression tree based models.	**3**	**4**	**7**	**8**	Solve assigned problems. Study Ch. 7 of [1] or alternative sources (references will be provided).
6. Solvency assessment of an insurance company in the EU. Risk and capital management of an insurance company.	**2**	**2**	**4**	**4**	Solve assigned problems. Read selected extracts from Solvency II legislation (references will be provided)
7. Discussion/ preparation of group assignments	**-**	**6**	**6**	**30**	Prepare group assignment and present to the class.
8. Final test	**2**	**-**	**2**	**15**	Revise the materials studied in class and home assignments.
Total (non-life insurance):	**20**	**24**	**44**	**85**	
1. Main features of pension systems. Three pension pillars. Classification of pensions. Necessity of solidarity between generations. Pension systems in European Union and USA.	2		2	16	To read assigned paper. To prepare and present short presentation (5-10 min.) about specific countries'pension system
2. Demographical models of pension systems: pure and relative probabilities; Lexis diagram. Funding models of pension systems: terminal funding; funding methods; individual / aggregate funding methods.	4	8	12	14	To read assigned material (demographical – financial models). To prepare for discussions.
3. Pension annuities: deterministic vs. stochastic approach. Longevity risk: cohort and period mortality tables. Projected mortality tables and methods	4	8	12	14	To read assigned material (pension annuities; projected mortality tables;

of projection: extrapolation; single entry mortality table (age adjustment method); parametric methods; Lee Carter method.					Lee Carter method). To prepare for discussions.
4. Longevity risk: coefficient of variation and other risk measures; risk and mortality scenario; pooling and non-pooling parts of risk. Methods for management of Longevity risk: hedging, reinsurance, longevity bonds and others.	8	8	16	14	To read assigned material (longevity risk; methods for longevity risk management). To prepare for discussions.
5. Final test	2		2	20	To prepare for exam. To revise theory and its applications
Total (pension systems):	**20**	**24**	**44**	**78**	
	36	**48**	**88**	**163**	

Assessment strategy	**Weight**	**Time of assessment**	**Criteria**
General assessment strategy. Final mark (final course mark) is average of final marks scored for each part (Non-life insurance and Pension systems). Final mark is rounded according to standard rules, e.g., if average is 9,5, final mark is 10; if average is 9,4, final mark is 9, etc. It is considered that student successfully accomplished the course if final course mark (after rounding) is no less than 5 (five) and final mark for each part separately is no less than 5, otherwise debt is assigned to student (course is not accomplished successfully). Final mark for each part is calculated as weighted averaged of marks given for individual / group assignments and final test. Final test for both parts is organized at the same time, however, marks are given separately. Alternatively joint assignment covering material from both parts may be given. In such case weight assigned to each course part is specified during final test.			
Evaluation of the individual or group assignments (1-4 during semester for each part)	**60%**	During the semester	Marks – 0; 2,5; 5; 7,5; 10 are given for each assignment. **Summative** assessment – average mark multiplied by weight. Quality of fulfilment of assignment, interpretation of results and answers to questions is assessed. Marks are given according to the following scheme: **10**: assignment was carried out without mistakes; interpretation of results was correct; all questions answered exhaustively and correctly **7,5**: some nonessential mistakes when performing assignment and / or when interpreting the results; or no less than 75% of questions answered correctly **5**: mistakes when performing assignment and / or when interpreting the results; less than 75% and no less than 50% questions answered correctly **2,5**: serious (essential) mistakes when performing assignment and / or when interpreting the results; less than 50% and no less than 25% questions answered correctly **0**: assignment was not carried out or performed but with many very serious essential mistakes; less than 25% of questions answered correctly

Final test	**40%**	At the end of the semester	In this exam, students are tested on the material from the semester. Exam may consist of combination of multiple-choice questions, problem solving questions and case study questions. For each question the allocation of available marks will be provided. The answers to multiple choice questions and problem-solving questions are assessed based on the number of correct answers and / or according to the scheme described in "Evaluation of the individual or group assignments". The answer to case study question is assessed by granting points per each relevant idea stated and briefly explained in the case study. Maximum score for final test is 10 (for each part separately). **Summative** assessment: average of final score for each part multipled by weight.

Author	Publication year	Title	Volume and/or publication number	Publication place and publisher
Required reading				
1. P.J. Boland	2007	Statistic and Probabilistic Methods in Actuarial Science	-	Chapman&Hall/CRC
2. A. Skučaitė	-	Pension Funds (Lecture notes)	-	Updated every year
3. Pitacco, E., et. al.	2009	Modelling Longevity Dynamics for Pensions and Annuity Business	-	Oxford University Press
Recommended reading				
4. D. Hindley	2018	Claims Reserving in General Insurance	-	Cambridge University Press
5. M.V. Wuthrich & C. Buser	2020	Data analytics for non-life insurance pricing	-	Swiss Finance Institute https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2870308
6. P. McCullagh & J.A. Nelder	1989	Generalized Linear Models 2nd ed.	-	Chapman&Hall/CRC
7. Barr, N.	2002	Reforming Pensions: Myths, Truths and Policy Choices	International Social Security Review, Vol. 55 2 /2002	Blackwell Publishers
8. Werding, M.	2003	After Another Decade of Reform: Do Pension Systems in Europe Converge?	CESifo DICE Report 1/2003	http://www.ifo.de/portal/pls/portal/docs/1/1193630.PDF