COURSE UNIT (MODULE) DESCRIPTION

Course unit (module) title	Code
Functional data analysis	

Lecturer(s)	Department(s) where the course unit (module) is delivered
Coordinator: prof. Alfredas Račkauskas **Other(s): Jovita Gudan**	Faculty of Mathematics and Informatics

Study cycle	Type of the course unit (module)
Second	Compulsory

Mode of delivery	Period when the course unit (module) is delivered	Language(s) of instruction
Face-to-face	Spring/autumn semesters	English/Lithuanian

Requirements for students	
Prerequisites: Knowledge of the basic principles of statistical inference is desirable, as well as some knowledge of the R computing environment	**Additional requirements (if any):** No

Course (module) volume in credits	Total student's workload	Contact hours	Self-study hours
10	300	100	200

Purpose of the course unit (module): programme competences to be developed
The course aims to introduce the key mathematical concepts and results that are relevant for the theoretical and practical development of the statistical analysis of data obtained from observations of stochastic processes. Programme competences: 1 (1.1, 1.2, 1.3, 1.4), 2 (2.1, 2.2, 2.3), 3 (3.2, 3.3, 3.4), 4 (4.1, 4.2), 6 (6.2, 6.3), 7 (7.1, 7.2, 7.3)

Learning outcomes of the course unit (module): after completion of the course students	Teaching and learning methods	Assessment methods
will understand both technical and conceptual aspects dealing with data treated as samples of functions, will know how to represent data as functions,	theoretical lectures and seminars, applicative R sessions	assessment of individual problem solutions, exam

will understand the role of functional data analysis to practical problems.		
will be able to exploit number of algorithmic approaches, with assistance of software packages, both to analyze and visualize functional data.	theoretical lectures and seminars, applicative R sessions	assessment of individual problem solutions, project completion and presentation, exam
will be able to make assessments of the type of approach that is most suitable for the problem at hand, and critically discuss the results of analysis obtained by a particular method.	research examples presented in seminars	project completion and presentation

Content: breakdown of the topics	Lectures and seminars	Tutorials	Computer sessions	**Contact hours**	**Self-study hours**	**Assignments**
Introduction to abstract statistics and statistics of random functions, examples and motivation (*objective is to provide students with an understanding of how functional data arises and the areas where it is commonly encountered, to present motivating examples as well as to provide a basic probabilistic structure for how functional data are enerated*)	6		4	**10**	**10**	To study [1, Ch. 1]; to solve homework assignments obtained during computer sessions
Smoothing and curve fitting (*objective is to demonstrate how one moves from observed vectors to functional objects, to discuss the basics of nonparametric smoothing (local polynomials, regression splines, smoothing splines, free-knot splines, wavelets), and in particular, to demonstrate*	6		4	**11**	**20**	To study [1,Ch. 3,4]; to solve homework assignments obtained during computer sessions

how basis expansions can be used to create and store functional objects).						
Registration of functional data	4		4	**9**	**10**	To study [1, Ch. 8], to solve homework assignments obtained during computer sessions
Clustering of functional data	4		2	**7**	**10**	to solve homework assignments obtained during computer sessions
Hilbert space framework for functional data (*objective is to provide a basic introduction to function spaces and operator theory, to discuss how functional objects can be modelled using this framework, and in particular, explore the basics of probability theory for function space valued objects (including distributions and other parameters)*).	8			**8**	**10**	To study [2, Ch. 3]
Descriptions of Functional Data. (*objective ate to learn how to estimating mean functions, covariance operators and other distributional parameters of functional data*).	4		2	**6**	**10**	To study [1, Ch. 6]; to solve homework assignments obtained during computer sessions
Dimensional reduction for functional data (functional principal component analysis, functional independent component analysis, varimax rotations) (*objectives are to introduce functional principal component analysis (FPCA) as one of the fundamental tools of FDA; to discuss the connections between FPCA, the spectral Theorem for covariance operators, and the Karhunen--Loéve expansion; to provide estimation methods and their properties*).	8		4	**13**	**20**	To study [1, Ch. 7] to solve homework assignments obtained during computer sessions
Non-parametric inference for functional data (*objectives are to introduce permutational inference, global and local null hypothesis testing, functional t-test, functional ANOVA;*	6		4	**10**	**20**	To study [2, Ch. 12] to solve homework assignments obtained during computer sessions

to provide their properties).						
Functional linear models (*objectives are to discuss the various forms of the functional linear model, including function on function regression or function/scalar regression; to present estimation methods and properties*).	6		4	**10**	**20**	To study [1, Ch. 9, 10] to solve homework assignments obtained during computer sessions
Dependent functional data (*objectives are to discuss generalizations to dependent sequences, including functional time series and space-time functional data*).	4		4	**8**	**10**	To study [2, Ch. 8] to solve homework assignments obtained during computer sessions
Project presentations *(Students may either investigate an FDA research paper/methodology not described in the class or to present a novel data application where they apply FDA methods*).	8			**8**	**40**	to work on a project;
Total	**64**	**0**	**32**	**96**	**170**	
Exam	2	2		**4**	**30**	Review theory and problem solutions; prepare for the exam
Total including final exam	**66**	**2**	**32**	**100**	**200**	

Assessment strategy	**Weight, %**	**Deadline**	**Assessment criteria**
Project/Presentation *Each student will work on a project throughout the semester.* *A final report will be submitted at the end of the semester and students will give a 15-20 minute presentation based on that report.*	40%	A final report will be submitted at the end of the semester	Students will give a 15-20 minute presentation based on the report. Marking scale: F - fail, C –threshold level, B – typical level, A –great level of achievements. A: 80-100 points/percent. A very good result with regard to theoretical depth, practical relevance, analytical ability and independent thought.

			B: 55-79 points/percent. The result is of a satisfactory standard with regard to theoretical depth, practical relevance, analytical ability and independent thought. C: 40-54 points/percent. The result satisfies the minimum requirements with regard to theoretical depth, practical relevance, analytical ability and independent thought, but not more. F: 0-39 points/percent. The result does not meet the minimum requirements with regard to theoretical depth, practical relevance, analytical ability and independent thought.
Homework *Homework will be assigned every second week. Students are encouraged to work together on homework, but each of them must turn in his/her own write up and answers.*	40%	Two weeks after homework assigned	Marking scale: F - fail, C –threshold level, B – typical level, A –great level of achievements. A: 80-100 points/percent. B: 55-79 points/percent. C: 40-54 points/percent. F: 0-39 points/percent
Final exam *Written closed book 2 hours duration exam.*	20%	during exam period	Marking scale: F - fail, C –threshold level, B – typical level, A –great level of achievements. A: 80-100 points/percent. B: 55-79 points/percent. C: 40-54 points/percent. F: 0-39 points/percent

Author	**Year of publication**	**Title**	**Publishing place and house or web link**
Compulsary reading			
J.O. Ramsay, G. Hooker and S. Graves	2009	Functional Data Analysis with R and MATLAB	Springer
Piotr Kokoszka and Matthew Reimherr	2017	Introduction to Functional Data Analysis	Taylor & Francis group

Optional reading			
Tailen Hsing and Randall Eubank	2015	Theoretical Foundations of Functional Data Analysis, with an Introduction to Linear Operators	Wiley
L. Horvath and P. Kokoszka	2012	Inference for functional data with applications	Springer
J. Ramsay and B. Silverman	2005	*Functional Data Analysis*, Second Edition.	Springer