Course description

Course title	Course code
Discrete market models	

Lecturer	Department where the course is delivered
Assoc. prof. Martynas Manstavičius	Department of Mathematical Analysis Faculty of Mathematics and Informatics Naugarduko St. 24, LT-03225 Vilnius, Lithuania

Cycle	Type of course
Second	Optional

Mode of delivery	Semester or period when the course is delivered	Language of instruction
Face-to-face	2nd semester (Spring)	Lithuanian, English

Prerequisites and corequisites	
Prerequisites: basic linear algebra and measure-theoretic probability theory	**Corequisites (if any) :**

Number of ECTS credits	Student's workload	Contact hours	Individual work hours
5	125	32	93

Course objectives: programme competences to be developed

The aim is to develop deeper understanding of the mathematical tools needed to model financial markets with and without friction, ability to analyze and value contingent claims (European and American put/call options, etc.) in discrete time setting, as well as ability to hedge risks associated with them by critically choosing an appropriate model. The course is intended for those who have not had a similar course during Bachelor studies.
Those who have already attended a similar course earlier will have an opportunity to deepen their knowledge of the subject by analyzing harder, more advanced topics, broadening application capabilities.

(Programme competences fostered: 1.1, 1.2, 3.2, 4.1, 6.1, 6.3)

Learning objectives	Learning methods	Assessment methods
- Ability to apply the principles of risk-neutral valuation including some versions of the No-arbitrage theorem	Traditional *lectures* to explain the theory of discrete time financial mathematics models. Occasional *Recitation classes* to solve problems that help understand the concepts and methods presented. *Individual work*: Solving complementary problems and studying the literature; participating in seminars, discussions For **advanced students** – prepare an exposition and seminar presentation – to strengthen research skills	Testing (open/closed book)
- Ability to model and price contingent claims in certain simple discrete time models with or without friction		
- Ability to validate the passage to the limit in the CRR model, leading to the Black-Scholes formula		
- Ability to appropriately organize various models, subject terminology, methods and conventions of mathematical finance in discrete time		Testing, written exam, presentation For **advanced students** – written exposition and seminar presentation
- Ability to estimate and hedge risks associated with financial derivatives in discrete time setting		

Course content: breakdown of the course	Contact hours					Individual work hours and assignments	
	Lectures	Consultations	Seminars	Recitation hours	**Total contact hours**	**Individual work hours**	**Assignments**
For students who have not attended a course on discrete time financial models							
1. Financial markets and traded assets	2				**2**	**6**	Read through, e.g., [6, Ch. I-II]
2. Single period model of a financial market	5				**5**	**10**	Read through [1, Ch. I.1-3], solve end-of-section problems, study recommended literature
3. Valuation of contingent claims. Complete and incomplete markets	2				**2**	**6**	Read through [1, Ch. I.4-5], solve end-of-section problems, study recommended literature
4. Risk and return	2				**2**	**6**	Read through [1, Ch. I.6], solve end-of-section problems, study recommended literature
5. Test 1 (preparation and writing)	2				**2**	**10**	Review theory and problem solutions [1, Ch. I.1–6]
6. Multiperiod model of a financial market	2				**2**	**6**	Read through [1, Ch. III.1-2], solve end-of-section problems, study recommended literature
7. Martingales and arbitrage-free market	5				**5**	**10**	Read through [1, Ch. III.3-4], solve end-of-section problems, study recommended literature
8. Binomial (CRR) model	5				**5**	**10**	Read through [1, Ch. III.5, IV.1-2], solve end-of-section problems, study recommended literature
9. Test 2 (preparation and writing)	2				**2**	**8**	Review theory and problem solutions [1, Ch. III.1-5, IV.1-2]
10. American options	5				**5**	**10**	Read through [1, Ch. IV.3-4], solve end-of-section problems, study recommended literature
11. Final exam (preparation and writing)						**13**	Review theory and problem solutions [1, Ch I, III-IV]
Total	**32**				**32**	**93**	
For advanced students who have attended a course on discrete time financial models earlier							
12. Hedging under constraints			**6**		**6**	**10**	Read [2, Ch. 7-9]; be ready for a discussion during the seminar, and/or prepare a presentation
13. Dalang-Morton-Willinger theorem			**24**		**24**	**70**	Thoroughly analyze [3,4] and prepare an exposition on the subject (follow the guidelines for term and final papers)
14. Presentation at a seminar			**2**			**13**	Prepare a presentation at a seminar; be ready to answer relevant questions

Assessment strategy	Weight	Time of assessment	Criteria
Tests 1&2 Each 2-hr test contains theoretical (closed-book) and problem solving (open book) parts. Points are awarded for each successfully answered question/problem. Test 1 contains material from topics I through V; Test 2 contains material from topics VI through VIII.	50% (25% each)	During classes (approx. during 6th and 12th week)	**Mark 10** – between 90% and 100% of available points on a test. **Mark 9** – between 80% and 89.99% of available points on a test. **Mark 8** – between 70% and 79.99% of available points on a test. **Mark 7** – between 60% and 69.99% of available points on a test. **Mark 6** – between 50% and 59.99% of available points on a test. **Mark 5** – between 40% and 49.99% of available points on a test. **Mark 1-4** – less than 40% of available points on a test.
Final exam The final 2-hr long written exam covers material from topics I through IV, VI through VIII, and X. It contains theoretical closed-book and practical open-book parts. Points are awarded for each successfully answered question/problem.	50%	During exam session	**Mark 10** – A student shows excellent knowledge of the course material, is able to analyze and generalize it, understands and correctly uses concepts, knows the main results of discrete time mathematical finance. He/she has collected between 90% and 100% of the available points. **Marks 8-9** – A student shows good/very good knowledge of the course material, is able to systematize and generalize it, understands used concepts, knows the majority of results of discrete time mathematical finance. 9 points are awarded for collecting between 80% and 89.99% of the available points; 8 points are awarded for collecting between 70% and 79.99% of the available points. **Marks 6-7** – A student understands the main concepts of the course and knows most of the main results of discrete time financial mathematics. 7 points are awarded for collecting between 60% and 69.99% of the available points; 6 points are awarded for collecting between 50% and 59.99% of the available points. **Mark 5 –** A student shows skin-deep understanding of the concepts of discrete time financial mathematics. He/she has collected between 40% and 49.99% of available points **Marks 1-4** – A student does not know the studied material and inappropriately uses the terms and concepts of the course. Has collected less than 40% of the available points.
For advanced students who have attended a course on discrete time financial models earlier			
Term paper	**80**	3-14th weeks	Based on the guidelines for preparation of term and final papers, write an exposition **(on the Dalang-Morton-Willinger theorem and its various proofs).** The grade will be based on mathematical correctness, precision, methodic presentation of concepts, statements and their proofs, complemented by examples and illustrations. A simple compilation of [3,4] is insufficient.
Presentation at a seminar	**20**	Approx.. 15-16th weeks	10-15 minute long presentation is graded based on style, mathematical consistency, ability to answer questions from the audience
The final grade is the sum of all collected grades multiplied by the corresponding weights and rounded to the nearest integer, e.g. , 8,5 is rounded upward to 9 and 8,49 is rounded downward to 8.			

Author	Publication year	Title	Volume and/or publication number	Publication place and publisher
Required reading				
1. S.R. Pliska	1997	Introduction to Mathematical Finance: Discrete Time Models		Oxford, Blackwell Publishers Inc.
2. H. Föllmer, A. Schied	2011	Stochastic finance. An introduction in discrete time	3rd edition	Berlin-New York, Walter de Gruyter
Additional reading				
3. F. Delbaen	1999	The Dalang-Morton-Willinger theorem		https://people.math.ethz.ch/~delbaen/ftp/teaching/DMW-Theorem.pdf (last checked 2020-01-28)
4. Y. Kabanov C. Stricker	2001	A teacher's note on no-arbitrage criteria		http://www.numdam.org/article/SPS_2001__35__149_0.pdf (last checked 2020-01-28)
5. J. Hull	2005	Options, futures and other derivative securities	6th ed.	Pearson Prentice Hall
6. R.E. Bailey	2005	The Economics of Financial Markets		Cambridge University Press, http://library.northsouth.edu/Upload/Economics%20Finance.pdf (last checked 2012-03-01)