

**DOCTORAL (PHD) STUDIES
COURSE DESCRIPTION**

Course title	Field of science	Faculty	Institute
Probability Theory and Mathematical Statistics	Mathematics (N 001)	Faculty of Mathematics and Informatics	Institute of Mathematics, Institute of Applied Mathematics
Study method	Number of credits	Study method	Number of credits
Lectures	2	Consultations	1
Individual work	7	Seminars	0

Course summary

I. Probability space.

Concept of probability space. Finite and discrete probability spaces. Construction of probability spaces on sets of real numbers, n-dimensional vectors, real valued sequences and functions.

II. Properties of probabilities.

The main probability properties. Conditional probabilities and their properties. Independent events.

III. Random variables.

Random variables. The main types of random variables. Transformations of random variables. Borel functions. Random vectors. Distribution functions.

IV. Expectation.

Expectation of random variable. Calculation of expectation. Properties of expectation. Moments of higher order and their properties.

V. Conditional expectation.

Conditional expectation with respect to a sigma algebra. Properties of conditional expectation. Calculation of conditional expectation.

VI. Characteristic functions.

Characteristic functions and their properties. Method of characteristic functions to consider weak convergence. Laws of large numbers. Central limit theorem. Central limit theorem. Central limit theorem. Infinitely divisible distributions. Stable distributions.

VII. Special probabilistic models.

Random walk and their main characteristics. Martingales and their properties. Method of martingales. Markov chain and their main characteristics. Stationary distributions of Markov chains.

VIII. Statistical models and their estimation.

Parametric models and methods of estimation. Estimators of parameters and their properties.

IX. Testing parametric hypotheses.

Statistic, null and alternative hypothesis, rejection area, significance level, p-values, test power. Uniformly most powerful tests.

X. Nonparametric statistics.

Chi-squared test and modifications. Examples or rank tests.

The number of contact hours: 64 academic hours.

Main literature

1. A.N. Shiryaev, *Probability*. Springer, 1995.

2. A.N. Shiryaev, *Probability-I*. Springer, 2016.

3. G.G. Roussas, *A Course in Mathematical Statistics*. San Diego, Calif: Academic Press, 1997, 2nd.

4. W.R. Pestman, I.B. Alberink. *Mathematical Statistics: Problems and Detailed Solutions*, De Gruyter, Inc., 1998.

5. Z. Govindarajulu, *Nonparametric Inference*, World Scientific Publishing Company, 2007.

Consulting teacher	Scientific degree	Pedagogical name	Main publications in the field of science of the last 5 year period
Jonas Šiaulyš	Ph.D. (HP)	Prof.	1. E. Bernackaitė, J. Šiaulyš , The finite-time ruin probability for an inhomogeneous renewal risk model. <i>Journal of Industrial and Management Optimization</i> , 2017, 13, 207-222.

			<ol style="list-style-type: none"> 2. S. Danilenko, J. Šiaulyš, G. Stepanauskas, Closure properties of O-exponential distributions. <i>Statistics and Probability Letters</i>, 2018, 140, 63-70. 3. O. Ragulina, J. Šiaulyš, Randomly stopped minima and maxima with exponential-type distributions. <i>Nonlinear Analysis: Modelling and Control</i>, 2019, 24, 297-313. 4. R. Leipus, J. Šiaulyš, On a closure property of convolution equivalent class of distributions. <i>Journal of Mathematical Analysis and Applications</i>, 2020, 490, 124226. 5. M. Dirma, S. Paukštys, J. Šiaulyš, Tails of the moments for sums with dominatedly varying random summands. <i>Mathematics</i>, 2021, 9, 824.
Rūta Levulienė	Ph.D.	Assoc. Prof.	<ol style="list-style-type: none"> 1. V. Bagdonavičius, R. Levulienė, Testing proportional hazards for specified covariates. <i>Modern stochastic: theory and applications</i>. Vilnius; Kiev : VTeX. ISSN 2351-6046. eISSN 2351-6054. 2019, vol. 6, no. 2, p. 209-225. 2. V. Bagdonavičius, R. Levulienė, On accelerated life testing when the AFT model fails. <i>IEEE transactions on reliability</i>. Piscataway : IEEE. ISSN 0018-9529. eISSN 1558-1721. 2019, vol. 68, iss. 4, p. 1311-1319. 3. V. Bagdonavičius, M.A. Hafdi, R. Levulienė, Modeling and analysis of data with confounding covariates and crossing of the hazard functions. <i>Communications in statistics - theory and methods</i>. Philadelphia : Taylor & Francis. ISSN 0361-0926. eISSN 1532-415X. 2021, vol. 50, no. 20, p. [5262-5284]. 4. J. Markevičiūtė, J. Bernatavičienė, R. Levulienė, V. Medvedev, P. Treigys, J. Venskus, . Attention-based and time series models for short-term forecasting of COVID-19 spread. <i>CMC-Computers, materials & continua</i>, ISSN 1546-2218. eISSN 1546-2226. 2021, first published online.

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Board Chairman – assoc. prof. dr. Kristina Lapin